3 MN 60702

B.Sc. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2024.

THIRD SEMESTER

Mathematics (Minor)

Course 5: GROUP THEORY

Time : Three hours | Maximum : 70 marks

(No additional sheet will be supplied)

PART A — ($5 \times 4 = 20$ marks)

Answer any FIVE of the following questions.

1. In a group G, inverse of any element is unique.

2. A finite semi group satisfying the cancellation laws is a group.

3. If H is any subgroup of a group G, then $H \bullet H = H$.

4. Prove that the union of two subgroups is subgroups if and only if one is contained in the other.

5. If a, b are any two elements of a group G and H any subgroup of G, then $a \in bH \Leftrightarrow aH = bH$.

6. If N, M are normal subgroups of G, then NM is also a normal subgroup of G.

7. Every homomorphic image of an abelian group is abelian.

8. If $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 2 & 4 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 1 & 2 & 5 \end{pmatrix}$ then find fg and gf.

9. Every cyclic group is an abelian group.

10. Show that the set of all cube roots of unity is a cyclic group w.r.t. multiplication.

PART B — ($5 \times 10 = 50$ marks)

Answer ALL of the following questions.

11. Prove that the set G of all real numbers other than '1' with operation $\oplus$ such that $a \oplus b = a + b - ab$ for $a, b \in G$ is an abelian group.

Or

12. Show that the fourth roots of unity form an abelian group w.r.t. multiplication.

13. If H and K are two subgroups of a group G, then HK is a subgroup of G iff $HK = KH$.

Or

14. State and prove Langrange's theorem for groups.

15. Prove that any two right cosets of H are disjoint or identical.

Or

16. If H is a normal subgroup of G. The set $\frac{G}{H}$ of all cosets of h in G w.r.t cosets multiplication is a group.

17. Every homomorphic image of a group G is isomorphic to some quotient group of G.

Or

18. The necessary and sufficient condition for a homomorphism f of a group G onto a group G' with Kernel K to be an isomorphism of G into G' is that $K = \{e\}$.

19. Every finite group G is isomorphic to a permutation group.

Or

20. Every subgroup of cyclic group is cyclic.
